திருவள்ளுவர் பல்கலைக்கழகம், வேலூர்
THIRUVALLUVAR UNIVERSITY, VELLORE

Ph.D., - COMMON ENTRANCE TEST (CET9) – JUNE SESSION 2022

Subject : **MATHEMATICS** Exam Date : **26.06.2022**

Time : 11.00 A.M. TO 12.30. P.M Maximum Marks : 50

NAME		**REGISTER NO**	
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SECTION – A (50 x 1 = 50 Marks)
All Questions carry equal marks

1. Let G be non-abelien group of order p^3 where p is a prime. If center of the group $Z(G)$ is not identity, then
   A) Order of $Z(G) = p$
   B) Order of $Z(G) = p^2$
   C) $\frac{G}{Z(G)}$ is cyclic
   D) None of the above

2. Let G be a group of order pqr, where p, q, r are primes and $p < q < r$. Which of the following statements are true?
   A) Sylow$r$ $-$subgroup of G is normal
   B) G has no normal subgroup of order qr
   C) Sylow$p$ $-$subgroup of Gneed not normal
   D) All of the above

3. Let R be a ring with unity such that each element of R is an idempotent. Then the characteristic of R is
   A) 0
   B) 2
   C) An odd prime
   D) 1

4. Let A and B be square matrices of order n. Then the minimum value of rank of (AB) is given by
   A) $rank(A) + rank(B)$
   B) $rank(A) + rank(B) + n$
   C) $rank(A) \times rank(B)$
   D) $rank(A) + rank(B) - n$

5. Let$A(\theta) = \begin{bmatrix} Cos\,\theta & -Sin\,\theta \\ Sin\,\theta & Cos\,\theta \end{bmatrix}$ $\theta \in (0,2\pi)$ Which of the following statements is ture ?
   A) $A(\theta)$ has eigenvectors in $\mathbb{R}^2$ for every $\theta \in (0, 2\pi)$
   B) $A(\theta)$does not have eigenvectors in in $\mathbb{R}^2$ for any $\theta \in (0, 2\pi)$
   C) $A(\theta)$ has eigenvectors in $\mathbb{R}^2$ for exactly one value of $\theta \in (0, 2\pi)$
   D) $A(\theta)$ has eigenvectors in $\mathbb{R}^2$ for exactly two value of $\theta \in (0, 2\pi)$

6. The sequence $n^{\frac{1}{n}}$ is
   A) monotonically decreasing
   B) monotonically increasing
   C) convergent and converges to zero
   D) neigher monotonically increasesing or monotonically decreasing

7. The value of the series $\sum_{n=1}^{\infty} \frac{n}{2^n}$ is given by
   A) 2
   B) 4
   C) 6
   D) 8

8. The set $A = \{x \in \mathbb{R}: xsin(x) \leq 1, xcos(x) \leq 1\}$ subset of $\mathbb{R}$ is
   A) Bounded closed set
   B) Unbounded closed set
   C) Bounded open set
   D) Unbounded open set

9. Let $\alpha = \lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2}$ and $\beta = \lim_{(x,y)\to(0,0)} \frac{(x^2-y^2)}{x^2+y^2}$.Then,
   A) α exists but β does not
   B) α does not exists but β exists
   C) α and β does not exist
   D) Both α, β does exist

10. The value of $\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$ is
   A) $\frac{1}{e}$
   B) 1
   C) e
   D) 0

11. The branch point of the Riemann Surface $w = z^n$ is
   A) $w = \pm\pi$
   B) $w = 0$
   C) $w = \pm 2\pi$
   D) $w = \pm 1$

12. If a mapping of Ω by $w = f(z)$ is topological then
   A) $z = f^{-1}(w)$ is also analytic
   B) $\bar{f}(z)$ is also analytic
   C) $f'(z_0) = 0$
   D) $f(z) = 0$

13. The level curves $u = u_0$ and $v = v_0$ of $w = z^2$ are
   A) Paraboloas
   B) Concentric circles
   C) Circles of Apollonius
   D) Equilateral hyperbolas

14. In the linear transformation of the form $w = k.\frac{z-a}{z-b}$ the point b in the z-plane corresponds to

A) $w = 0$
B) $w = z + c$
C) $w = b$
D) $w = \infty$

15. If a curve γ lies inside a circle and "a" does not pass through γ then with usual notation, $n(\gamma, a)$=_____

A) $n(-\gamma, a)$
B) 0
C) $-n(\gamma, a)$
D) 1

16. The two linearly independent solutions of y″ + y = 0 are

A) e^x , e^{-x}
B) e^{2x} ,e^{-x}
C) e^{ix}, e^{-ix}
D) e^{2ix} , e^{-2ix}

17. $\frac{dy}{dx} + Py = Q$is a linear differential equation of first order if

A) P, Q are functions of x only
B) P, Q are functions of y only
C) P, Q are functions of x and y
D) None of these

18. Let φ satisfy $\varphi(x) = f(x) + \int_0^x \sin(x-t)\,\varphi(t)\,dt$. Then φ is given by

A) $\varphi(x) = f(x) + \int_0^x (x-t) f(t) dt$
B) $\varphi(x) = f(x) - \int_0^x (x-t)\, f(t) dt$
C) $\varphi(x) = f(x) - \int_0^x \cos(x-t) f(t)\, dt$
D) $\varphi(x) = f(x) - \int_0^x \sin(x-t) f(t) dt$

19. The partial differential equation $x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial y^2} = 0$ is

A) Hyperbolic for$x > 0, y > 0.$
B) Elliptic for $x > 0, y < 0.$
C) Hyperbolic for $x > 0, y < 0$
D) Elliptic for $x < 0, y < 0.$

20. Let y_1 and $y_2$be two solutions of the problem $y''(t) + ay'(t) + by(t) = 0, t \in R, y(0) = 0$ where a and b are real constants. Let W be the Wronskian of y_1 and y_2. Then

A) W is a non constant positive function
B) $W(t) \;=\; a$ for all $t \in R$, for some c > 0
C) $W(t) \;=\; 0$ for all $t \in R.$
D) There exists $t_1, t_2 \in R$ such that $W(t_1) < 0 < W(t_2)$.

21. The assignment Problem is a

A) Non linear programming problem
B) Dyanamic programming problem
C) Integer linear programming problem
D) Integer non linear programming problem

22. Given that $(a, m) = d$ and $d|b$, then the linear congruence $ax \equiv b(mod\ m)$ has exactly
   A) One solution
   B) Solutions
   C) m solutions
   D) d solutions modulo m

23. Number of edges in a complete graph, K_n with n vertices
   A) $\frac{n(n+1)}{2}$
   B) $\frac{n(n-1)}{2}$
   C) n
   D) $n - 1$

24. The sum of the degrees of every vertex in a graph G is
   A) Always odd
   B) Always a power of 2
   C) Always even
   D) Can be any positive integer

25. Let G be a non-trivial connected graph. Then G contains an open Eulerian trail if and only if
   A) G has exactly two odd degree vertices
   B) All its vertices has even degree
   C) G has exactly one odd vertex
   D) None of the above

26. Let S be a nonempty proper subset of vertices of a Hamiltonian graph G and $\omega(H)$ denotes the number of components in any graph H, then
   A) $\omega(G - S) \leq |S|$
   B) $\omega(G - S) \geq |S|$
   C) $\omega(G - S) = |S|$
   D) None of the above

27. Identify a necessary condition for a continuously differentiable function $f(x_1, x_2 \dots x_n)$of n variables $x_1, x_2 \dots x_n$ to have a relative maximum or minimum at an interior point of a region.
   A) $df = \frac{\partial F}{\partial x_1}dx_1 + \frac{\partial F}{\partial x_2}dx_2 + \cdots \frac{\partial F}{\partial x_n}dx_n$
   B) $\frac{\partial F}{\partial x_1} = \frac{\partial F}{\partial x_2} = \cdots = \frac{\partial F}{\partial x_n} = 0$
   C) $\frac{dF}{dx_1} = \frac{dF}{dx_2} = \cdots = \frac{dF}{dx_n} = 0$
   D) $dF = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial y'}dy'$

28. Under usual notation, when $y(x_1)$ is not given, the natural boundary condition for $y(x)$ is
   A) $\left[\frac{\partial f}{\partial y'}\eta[x]\right]_{x=x_1} - \left[\frac{\partial f}{\partial y'}\eta[x]\right]_{x=x_2} = 0$
   B) $\left[\frac{\partial f}{\partial y'}\right]_{x=x_1}=0$
   C) $\left[\frac{\partial f}{\partial y'}\right]_{x=x_2}=0$
   D) $\left[\frac{\partial f}{\partial y'}\eta[x]\right]_{x=x_1} + \left[\frac{\partial f}{\partial y'}\eta[x]\right]_{x=x_2} = 0$

29. Given than y(x) has a discontinuity at c, the natural transition condition is represented by

A) $\lim_{x\to c^+} \frac{\partial f}{\partial y'} = \lim_{x\to c^-} \frac{\partial f}{\partial y'}$

B) $\lim_{x\to c^+} \frac{\partial f}{\partial y} = \lim_{x\to c^-} \frac{\partial f}{\partial y}$

C) $y(c^+) + y(c^-) = y(c)$

D) $y(c^+) - y(c^-) = y(c)$

30. A ball of mass 8gms moving with velocity 10cm/sec impinges directly on another of mass 24gms, moving at 2cm/sec in the same direction. If e = 0.5, then their respective velocities after impact is

A) 5cm/sec, 1cm/ sec

B) 5 cm/sec, 0cm/ sec

C) 1 cm/sec, 5cm /sec

D) 0 cm/sec, 5cm/ sec

31. If three equal forces each equal to p act along the sides of a triangle ABC taken in order then the algebraic sum of the resolved parts of the forces along BC is

A) $p(cosC - cos B)$

B) $p(1 - cos C - cos B)$

C) $p(cos C + cos B)$

D) $p(1 + \cos C + \cos B)$

32. Let (X, d) be a metric space where X is an infinite set and d is the discrete metric.

A) Heine – Borel theorem holds for (X, d)

B) Heine – Borel theorem does not holds for (X, d)

C) X is not bounded

D) $X$is compact

33. For $x = (x_1, x_2, x_3), y = (y_1, y_2, y_3) \in \mathbb{R}^3$

$$d_1(x, y) = \max_{1\leq j\leq 3} |x_j - y_j|$$

$$d_2(x, y) = \sqrt{\sum_{j=1}^{3} (x_j - y_j)^2}$$

Consider the metric spaces $(\mathbb{R}^3, d_1)$and $(\mathbb{R}^3, d_2)$, then

A) $(\mathbb{R}^3, d_1)$ is complete, but $(\mathbb{R}^3, d_2)$ is not complete

B) $(\mathbb{R}^3, d_2)$ is complete, but $(\mathbb{R}^3, d_1)$ is not complete

C) Both $(\mathbb{R}^3, d_1)$ and $(\mathbb{R}^3, d_2)$ are complete

D) Neither $(\mathbb{R}^3, d_1)$ nor $(\mathbb{R}^3, d_2)$ is complete

34. Let f be a function defined on $[0, \infty)$ by $f(x) = [x]$, the greatest Integer less than or equal to x. Then

A) $f$is continuous at each point of $\mathbb{N}$.

B) $f$is continuous on $[0, \infty)$.

C) f is continuous on $[0, 1]$

D) f is discontinuous at $x = 1, 2, 3, \ldots$

35. Which of the following statement is correct?

A) Every compact Hausdorff space is normal.

B) Every compact Tychonoff space is normal.

C) Every Hausdorff space is Regular.

D) None of the above.

36. Co-countable topology is finer than
   A) usual topology
   B) product topology
   C) co-finite topology
   D) none of the above

37. If X is Hausdorff then
   A) the complement of any finite set is closed.
   B) the complement of any finite set is open.
   C) the complement of any finite set is compact.
   D) All of the above.

38. For Bernoulli's equation to be applied, which of the following assumptions must be met?
   A) The flow must be steady.
   B) The flow must be incompressible
   C) Friction by viscous forces must be minima
   D) All of the above

39. In fluid flow, the line of constant piezometric head passes through two points which have the same ____________
   A) velocity.
   B) pressure.
   C) elevation.
   D) All of the above.

40. Rotameter is a device used to measure
   A) Absolute pressure
   B) Velocity of fluid
   C) Rotation
   D) Flow

41. An ideal flow of a liquid obeys
   A) Continuity equation
   B) Newton's law of viscosity
   C) Newton's second law of motion
   D) dynamic viscosity law

42. Trapezoidal rule gives exact value of the integral if the integrand is a
   A) Quadratic function
   B) Continuous function
   C) linear function.
   D) None of the above

43. Order of convergence of Newton Raphson method is
   A) 1
   B) 2
   C) 3
   D) 4

44. If $f(0) = 3, f(1) = 5, f(3) = 21$, then the unique polynomial of degree 2 or less using newton divided difference interpolation will be
   A) $2x^2$
   B) $2x^2 + 1$
   C) $2x^2 + 3$
   D) x^{2+2}

45. Whichofthefollowingtestis usedtotesttheequalityoftreatmentmeansinANOVA?

A) x^2-test
B) t-test
C) F-test
D) standard normal

46. What is the expected number of heads appearing when a fair coin is tossed three times?

A) 2.2
B) 2
C) 1
D) 1.5

47. Which of the following principles of experimental design is/are used in CRD?

A) randomizationonly
B) randomization andreplication
C) replicationonly
D) localcontrolandrandomization

48. In estimating population mean based on a stratified sample with maximum precision for afixed cost, take alarge sample from astratumif

A) the stratum is larger
B) samplingischeaperinthestratum
C) thestratumismorevariableinternally
D) conditionsineither(A),(B)and(C)orallsimultaneouslyhold

49. Let X_i's be independent random variables such that X_i's are symmetric about 0 and $Var\,(X_i) = 2i-1$, for $i \geq 1$. Then, $\lim_{n\to\infty} P(X_1 + X_2 + \cdots + X_n > n\,logn)$

A) Equals 0
B) Equals $\frac{1}{2}$
C) Equals 1
D) Equals 3

50. Three unbiased coins are tossed. What is the probability of getting at most two heads

A) $\frac{1}{3}$
B) $\frac{1}{9}$
C) $\frac{7}{8}$
D) $\mathbb{P}(E) = \frac{5}{8}$